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Purely Cubic Action for Superstring Field via Noncommutative Supergeometry

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Witten's superstring field action is shown to be obtained from a purely cubic action by using noncommutative supergeometry.

It has been shown by Horowitz *et al.* (1986) that the background dependence of Witten's (1986*a*) open bosonic string field theory action can be eliminated in the purely cubic action. In this paper we will show that this procedure can be applied to the open superstring field theory (Witten, 1986*b*) by using noncommutative supergeometry. Consider the noncommutative superalgebra $(S, \circ)$ consisting of two noncommutative algebras $(\Psi, \bullet)$ and $(A, *)$, where Ψ (resp. A) means the algebra induced by odd parity (resp. even parity). Then we can decompose S into a direct product of two noncommutative algebras

$$S = A \times \Psi \tag{1}$$

Since Witten's superstring field theory is manifestly space-timely supersymmetric, it is obvious that the dimension of A is equal to that of Ψ. Therefore, in the course of our study, we will assume the existence of a natural homeomorphism $\xi: \Psi \to A$. Let (a, ψ), $(a', \psi') \in S$; then the multiplication $\circ$ in S can be designed in the following way

$$(a, \psi) \circ (a', \psi') = (a'', \psi'') \in S \tag{2}$$

The most suitable ansatzes for a'' and ψ'' are then given by

$$a'' = \xi(\psi \bullet \psi') + h(a * a') \tag{3}$$

$$\psi'' = k(\psi \bullet \xi^{-1}(a') + \xi^{-1}(a) \bullet \psi') \tag{4}$$

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where k and h mean the homeomorphisms from $\Psi \to \Psi$ and $A \to A$, respectively. From the associativity of Ψ and A, it follows that S is associative iff, for $\forall a, b, c \in A$, $\forall \psi, \psi' \in B$,

$$\xi k \xi^{-1} = h \tag{5}$$

$$h((a * b) * c) = h(a * b) * c \tag{6}$$

$$h(a * (b * c)) = a * h(b * c) \tag{7}$$

$$\xi(\psi \bullet \psi') = \xi(\psi) * \xi(\psi') \tag{8}$$

$$k(\xi^{-1}(a * b) \bullet \psi) = \xi^{-1}(h(a * b)) \bullet \psi \tag{9}$$

$$k(\psi \bullet \xi^{-1}(a * b)) = \psi \bullet \xi^{-1}(h(a * b)) \tag{10}$$

Now consider the superintegration map

$$\oint: \quad S \to C \tag{11}$$

where C is a set of complex numbers. Since $\Psi \cap C = 0$, we have

$$\oint (0, \psi) = 0 \tag{12}$$

In order to define $\oint(a, 0)$, we should investigate the ghost number of each operator and field. Consider the free action

$$\oint (a, \psi) \circ Q(a, \psi) \tag{13}$$

where

$$Q(a, \psi) = (Q_1 a, Q_2 \psi) \tag{14}$$

Here Q_1, Q_2 mean the BRST operator in A, Ψ space, respectively. This action has the gauge invariance

$$(\delta a, \delta \psi) = (Q_1 \lambda; Q_2 \chi) \tag{15}$$

Denoting the ghost number by $[\cdots]$, Witten (1986*b*) shows that

$$[a] = -1/2$$

$$[\psi] = 0$$

$$[Q_1] = [Q_2] = 0$$

$$[\lambda] = -3/2$$

$$[\chi] = -1$$

From the algebras

$$(\lambda, 0) \circ (\lambda', 0) = (h(\lambda * \lambda'), 0)$$
$$(\lambda, 0) \circ (0, \chi) = (0, k(\lambda \bullet \xi^{-1}(\chi))$$
$$(0, \chi) \circ (0, \chi') = (0, \xi(\chi \bullet \chi))$$

we have

$$[h] = 1$$
$$[k] = 1$$
$$[\xi] = [\xi^{-1}] = 0$$

where we used the result of Witten (1986*b*),

$$[*] = [\bullet] = 1/2$$

Since the free Lagrangian density has the ghost number 3/2, the integration operator for noncommutative supergeometry should have the ghost number $-3/2$. However, the usual integration ($\int$) has the ghost number $-1/2$; we must insert some operator with ghost number -1. According to the discovery of Friedan *et al.* (1986),

$$[h^{-1}] = [k^{-1}] = -1$$

where

$$hh^{-1} = h^{-1}h = I_1$$
$$kk^{-1} = k^{-1}k = I_2$$

Therefore we can define the superintegral in the following way:

$$\oint (a, \psi) = \int h^{-1}a \tag{16}$$

Summarizing all the results, we have the following noncommutative superalgebra:

$$\oint (a, \psi) \tag{17}$$

$$[(a_1, \psi_1) \circ (a_2, \psi_2)] \circ (a_3, \psi_3) = (a_1, \psi_1)[\circ (a_2, \psi_2) \circ (a_3, \psi_3)] \tag{18}$$

$$Q[(a, \psi) \circ (a', \psi')] = Q(a, \psi) \circ (a', \psi') + (-)^{(a,\psi)}(a, \psi) \circ Q(a', \psi') \tag{19}$$

$$\oint (a, \psi) \circ (a', \psi') = (-)^{(|a||a'|,|\psi||\psi'|)} \oint (a', \psi') \circ (a, \psi) \tag{20}$$

From (3)–(5), the purely cubic action is then given by

$$S_c[\psi, a] = \int [h^{-1}\xi(\psi \bullet k(\psi \bullet \xi^{-1}(a) + \xi^{-1}(a) \bullet \psi)) + a * \xi(\psi \bullet \psi) + a * h(a * a)] \tag{21}$$

The equations of motions are

$$a * a = 0 \tag{22}$$

$$\psi \bullet \psi = 0 \tag{23}$$

The special solution for (22) and (23) is

$$(a_0, \psi_0) = (h^{-1}Q_{1L}I_1, 0) \tag{24}$$

Here Q_L was first introduced in Horowitz *et al.* (1986) to obtain the purely cubic action for the bosonic open string and satisfies the following relations:

$$QI = (Q_L + Q_R)I = 0 \tag{25}$$

$$Q_R A * B = -(-)^A A * Q_L B \tag{26}$$

$$Q_R\psi \bullet \psi' = -(-)^{\psi}\psi \bullet Q_L\psi' \tag{27}$$

$$[Q, Q_L]_+ = 0 \tag{28}$$

where Q_L (Q_R) mean the left (right) part of BRST charge. Perturbating the string field around this special solution

$$a = h^{-1}Q_{1L}I_1 + a' \tag{29}$$

$$\psi = \psi' \tag{30}$$

we have

$$S_c[a, \psi] = S_c[a', \psi'] - 3\int h^{-1}\xi(\psi' \bullet Q_{2L}I_2 \bullet \psi') - 3\int a' * Q_{1L}I_1 * a' \tag{31}$$

where we used the relations (5)–(10), (20), and

$$\xi^{-1}(Q_{1L}I_1) = Q_{2L}I_2 \tag{32}$$

On the other hand, we have

$$\int a' * Q_{1L}I_1 * a' = \frac{1}{2}\int a' * (Q_{1L}I_1 * a' - a' * Q_{1L}I_1) = \frac{1}{2}\int a' * (-Q_1)a' \tag{33}$$

where we used the relations (25), (26), and (28). Similarly, we have

$$\int h^{-1}\xi(\psi' \bullet Q_{2L}I_2 \bullet \psi') = \frac{1}{2}\int h^{-1}\xi\psi' \bullet (-Q_2)\psi' \tag{34}$$

We have obtained Witten's superstring field theory action from the purely cubic action by using noncommutative supergeometry.

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